Business Statistics: A Decision-Making Approach

6th Edition

Chapter 7

Estimating Single Population Parameters

Chapter Goals

After completing this chapter, you should be able to:

- Distinguish between a point estimate and a confidence interval estimate
- Construct and interpret a confidence interval estimate for a single population mean using both the z and t distributions
- Determine the required sample size to estimate a single population mean within a specified margin of error
- Form and interpret a confidence interval estimate for a single population proportion

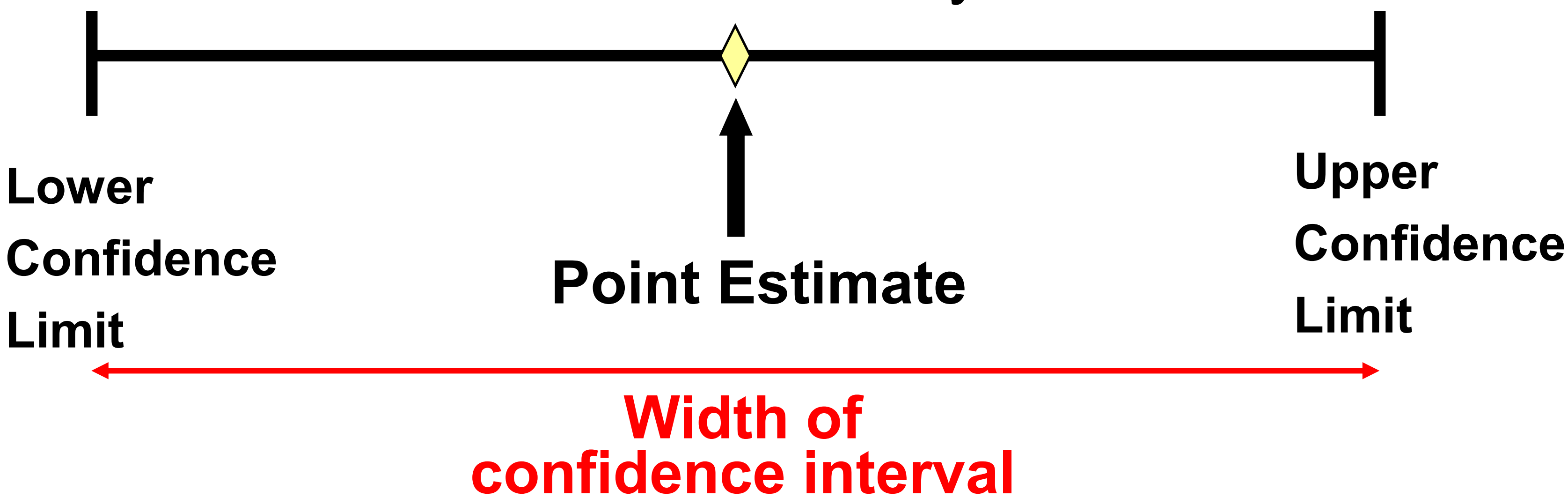
Confidence Intervals

Content of this chapter

- Confidence Intervals for the Population Mean, μ
  - when Population Standard Deviation σ is Known
  - when Population Standard Deviation σ is Unknown
- Determining the Required Sample Size
- Confidence Intervals for the Population Proportion, p

Point and Interval Estimates

- A point estimate is a single number, used to estimate an unknown population parameter
- a confidence interval provides additional information about variability



Point Estimates

We can estimate a Population Parameter …		with a Sample Statistic (a Point Estimate)
Mean	μ	$\bar{X}$
Proportion	π	p
Standard Deviation	σ	s

Confidence Intervals

- How much uncertainty is associated with a point estimate of a population parameter ?
- An interval estimate provides more information about a population characteristic than does a point estimate
- Such interval estimates are called confidence intervals

Confidence Interval Estimate

- An interval gives a range of values:
  - Takes into consideration variation in sample statistics from sample to sample
  - Based on observation from 1 sample
  - Gives information about closeness to unknown population parameters
  - Stated in terms of level of confidence
    - Never 100% sure

Confidence Intervals

- The favored form of estimates in formal reports of different studies

- An interval estimate of a population parameter is an interval of the form:

$$\hat{\theta}_L < \theta < \hat{\theta}_U$$

- Where $\hat{\theta}_L$ and $\hat{\theta}_U$ depend on the value of the statistic $\hat{\theta}$ for a particular sample and also the sampling distribution of $\hat{\theta}$

Confidence Intervals

- For example:

$$\hat{\mu}_L < \mu < \hat{\mu}_U$$

depends on $\bar{X}$ and its sampling distribution

- The reported value of the population parameter (μ , σ , π) is inexact, and therefore the need for limits in the inequality which provide the endpoints for the interval estimate

Confidence Intervals

- Several methods are available for expressing the interval for waiting time:
  - $35 \leq \mu \leq 45$ seconds
  - $\mu = [35, 45]$ seconds
  - $\mu = 40 \pm 5$ seconds
- In either cases, the central value (40 in the above example) is often the computed value of the sample statistic serving as the estimator

Confidence Intervals

- The quality of a statistical estimate is measured in two dimensions:
  - Accuracy or Precision
  - Reliability
- The limits of the interval estimate indicate the degree of accuracy or precision of the point estimate. A more accurate estimate of the mean waiting time above would be:
  - $39 \le \mu \le 41$ seconds
  - $\mu = 40 \pm 1$ seconds

Confidence Intervals

- Reliability of the estimate is simply the probability that it is correct

$$P\left(\hat{\theta}_L < \theta < \hat{\theta}_U\right) = 1 - \alpha$$

- The interval $\hat{\theta}_L < \theta < \hat{\theta}_U$ computed from the selected sample is then called a (1 – α) 100% confidence interval level
- The fraction 1 – α is called the confidence coefficient or the degree of confidence

Confidence Level

- Confidence Level
  - Confidence in which the interval will contain the unknown population parameter
- A percentage (less than 100%)

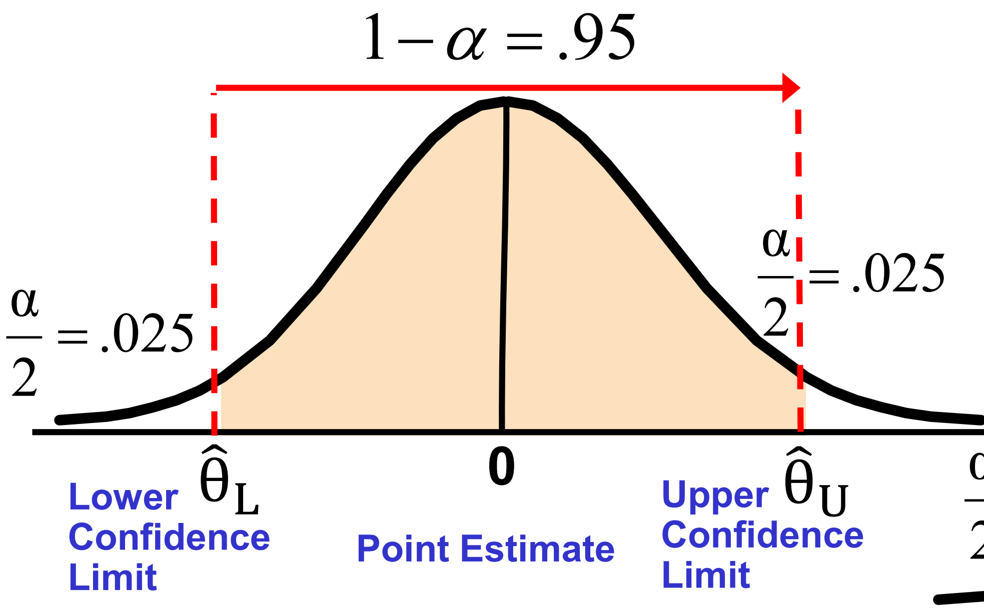
Confidence Level, $(1-\alpha)$

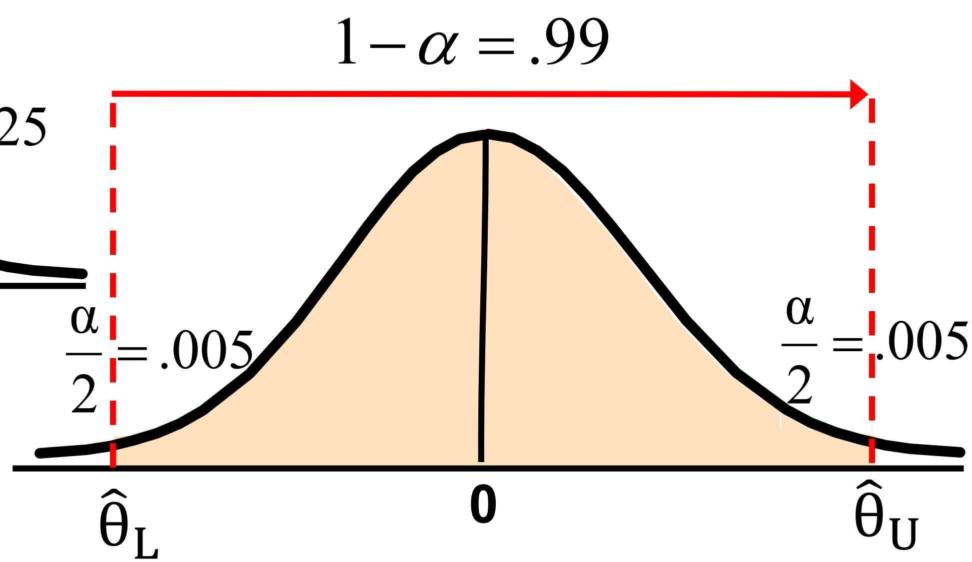
(continued)

- Suppose confidence level = 95%
- Also written $(1 - \alpha) = 0.95$
- A relative frequency interpretation:
  - In the long run, 95% of all the confidence intervals that can be constructed will contain the unknown true parameter
- A specific interval either will contain or will not contain the true parameter
  - No probability involved in a specific interval

Confidence Level

- <u>Ex</u>: when α = 0.05 we have a 95% confidence interval, and when α = 0.01 we have a 99% confidence interval

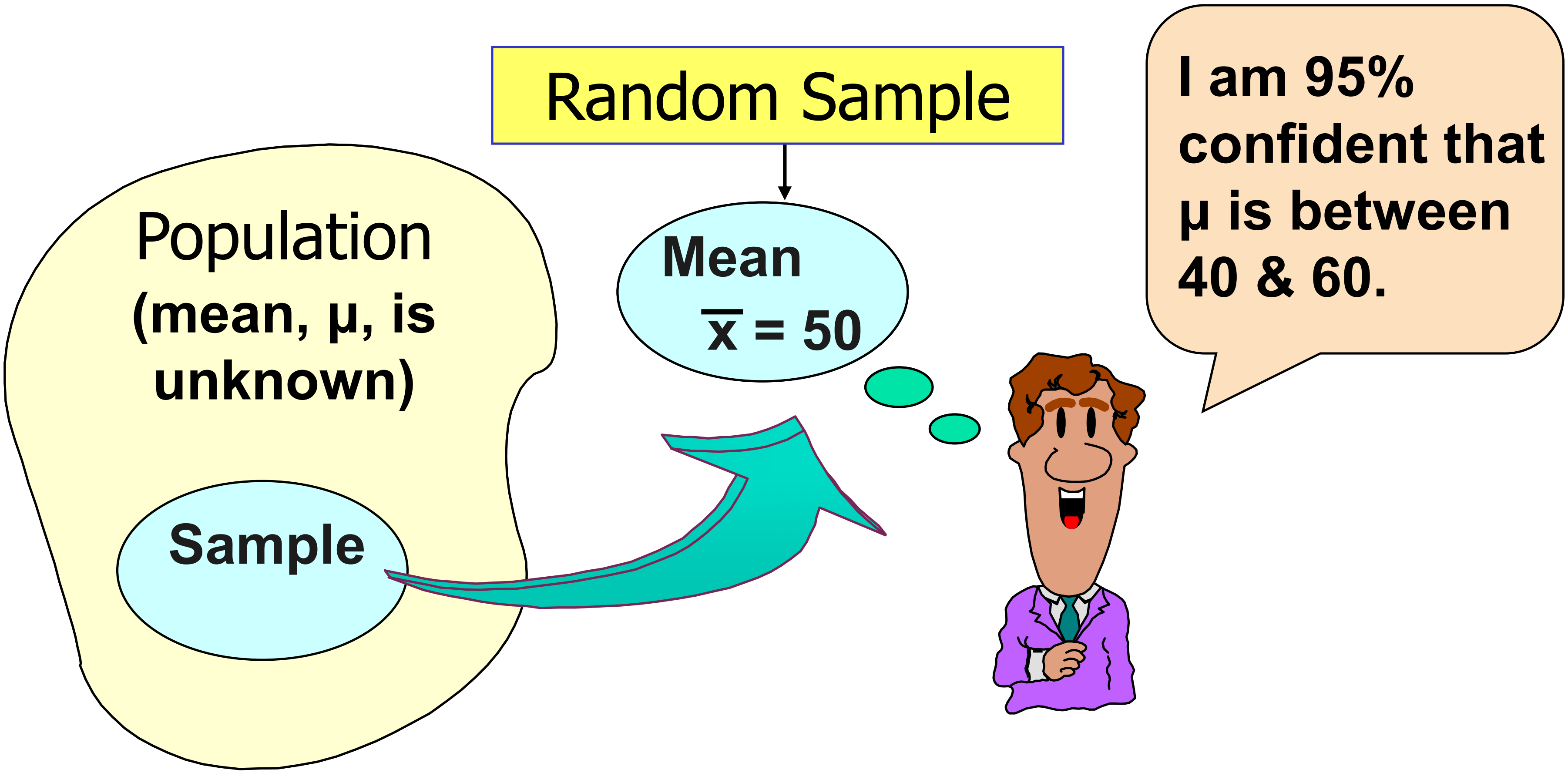




Confidence Level

- Reliability and accuracy are competing ends
- One can always improve at the expense of the
  - The higher the reliability , the wider and the less accurate the interval becomes
  - The higher the accuracy , the lower the reliability and the narrower the interval becomes
- Ex: It is better to be 95% confident that the average life of a certain TV transmitter is between 6 and 7 years than to be 90% confident that it is between 3 and 10 years

Estimation Process

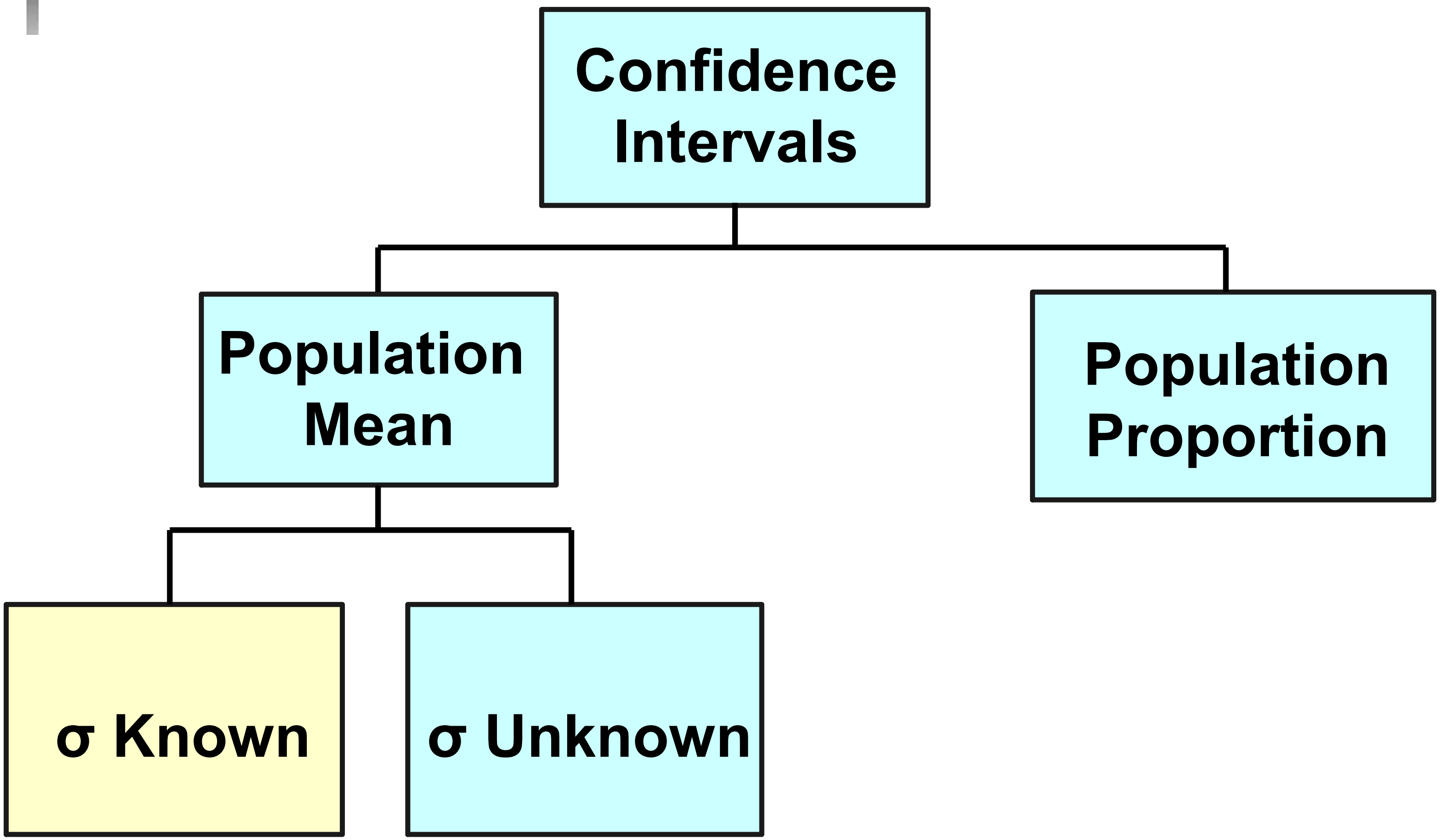


General Formula

- The general formula for all confidence intervals is:

Point Estimate ± (Critical Value)(Standard Error)

Confidence Intervals



Confidence Interval for μ (σ Known)

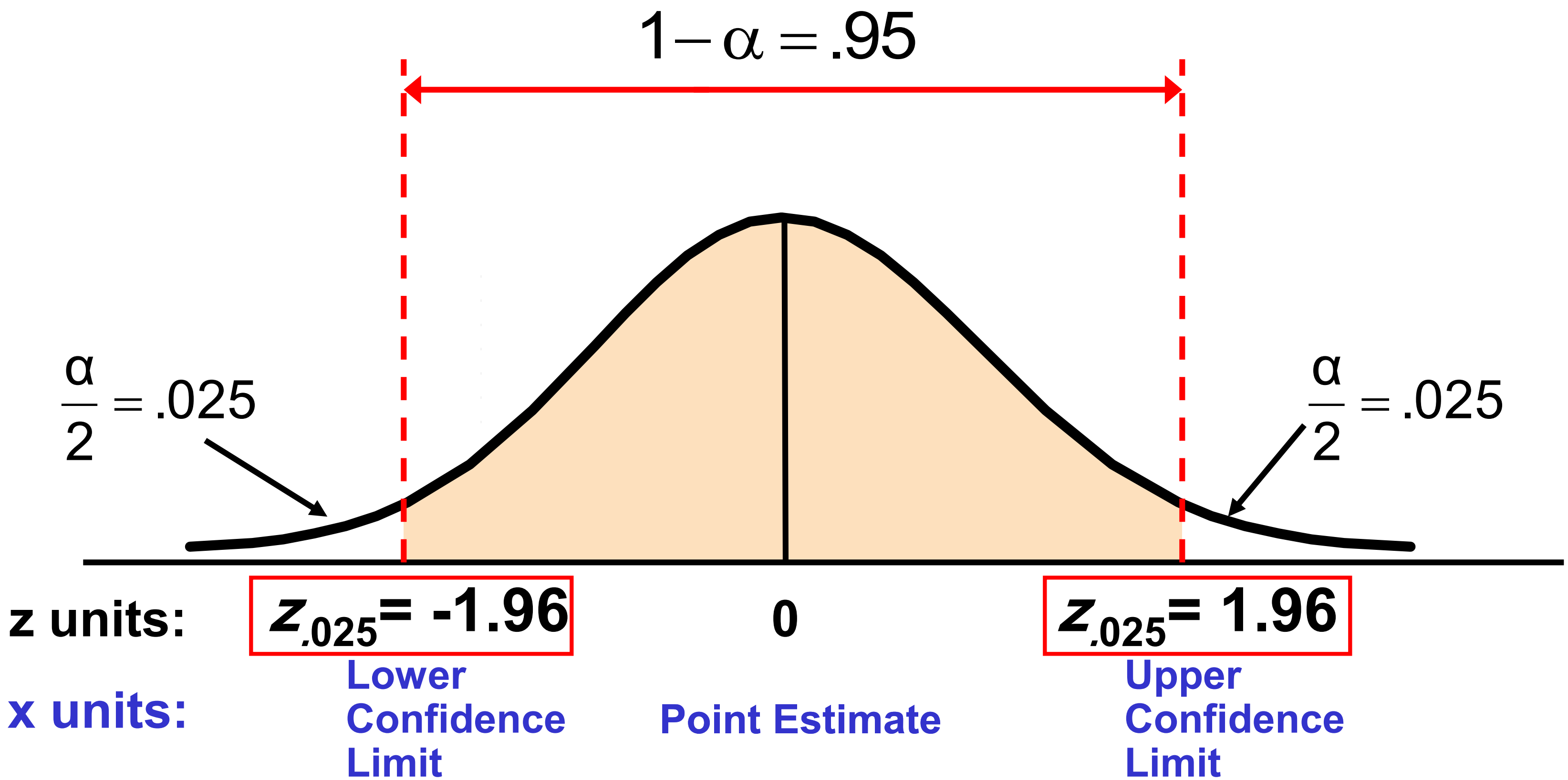
- Assumptions
  - Population standard deviation σ is known
  - Population is normally distributed
  - If population is not normal, use large sample

- Confidence interval estimate

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Finding the Critical Value

- Consider a 95% confidence interval: $\boxed{z_{\alpha/2} = \pm 1.96}$



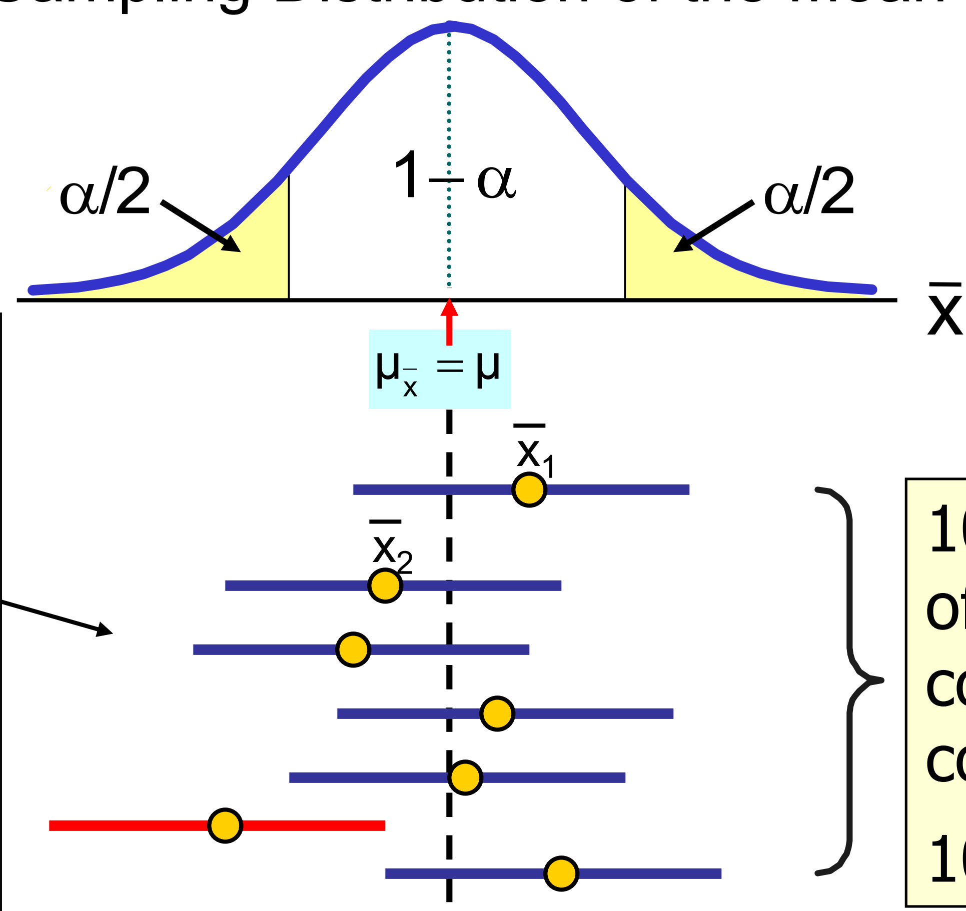
Common Levels of Confidence

- Commonly used confidence levels are 90%, 95%, and 99%

Confidence Level	***Confidence Coefficient,*** $1-\alpha$	***z value,*** $Z_{\alpha/2}$
80%	**.80**	**1.28**
90%	**.90**	**1.645**
95%	**.95**	**1.96**
98%	**.98**	**2.33**
99%	**.99**	**2.57**
99.8%	**.998**	**3.08**
99.9%	**.999**	**3.27**

Interval and Level of Confidence

Sampling Distribution of the Mean



Intervals extend from

$$\bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

to

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$100(1-\alpha)\%$ of intervals constructed contain μ;

$100\alpha\%$ do not.

Confidence Intervals

Interval & Level of Confidence

- There are 2 ways to improve the situation:
  - Raise the confidence level, using a bigger z (and reducing the accuracy)
  - Increase the sample size

Margin of Error

- Margin of Error (e): the amount added and subtracted to the point estimate to form the confidence interval

Example: Margin of error for estimating μ, σ known:

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \qquad e = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Factors Affecting Margin of Error

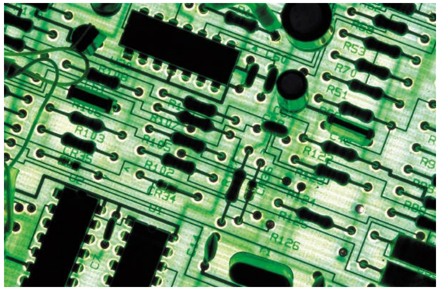
$$e = z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

- Data variation, σ : e ⇩ as σ ⇩
- Sample size, n : e ⇩ as n ⇧
- Level of confidence, 1 - α : e ⇩ if 1 - α ⇩

Example

- A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is .35 ohms.

- Determine a 95% confidence interval for the true mean resistance of the population.



Example

(continued)

- A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is .35 ohms.

- Solution:

$$\mu = \bar{X} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$= 2.20 \pm 1.96\,(.35/\sqrt{11})$$

$$= 2.20 \pm .2068$$

$$\boxed{1.9932 < \mu < 2.4068}$$